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空间扩散方程 Cauchy 问题的解及其正规性

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摘　要: 文章研究了三维扩散方程 Cauchy 问题。使用变数替换方法得到一个形式解表达式,最后给出了这个形式解是正规解的一个条件。此结果有利于理论分析和应用。

关键词: 扩散方程;Cauchy 条件;Fourier 变换;Laplace 变换;正规性

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对于三维无限区域扩散方程 Cauchy 问题

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}-a^2\left(\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}\right)=f(x,y,z,t)\\ &u|_{t=0}=g(x,y,z)\\ &((x,y,z)\in R^3,0<t<+\infty)\end{aligned}\right\} \quad (1)$$

其中:a 是正数;R 是实数域。如果其中函数 $f(x,y,z,t)$ 与 $g(x,y,z)$ 均分别关于 x,y,z 存在 Fourier 变换,$f(x,y,z,t)$ 关于 t 存在 Laplace 变换,则使用这些变换,可求得该问题的形式解的积分表达式[1-8]:

$$u=u(x,y,z,t)=\left(\frac{1}{2a\sqrt{\pi t}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}g(x_0,y_0,z_0)\times$$
$$\exp\left[-\frac{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}{4a^2t}\right]\mathrm{d}x_0\mathrm{d}y_0\mathrm{d}z_0+$$
$$\left(\frac{1}{2a\sqrt{\pi}}\right)^3\int_0^t\left(\frac{1}{\sqrt{t-\tau}}\right)^3\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x_0,y_0,z_0,\tau)\times$$
$$\exp\left[-\frac{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}{4a^2(t-\tau)}\right]\mathrm{d}x_0\mathrm{d}y_0\mathrm{d}z_0 \quad (2)$$

然而,该形式解的表达式,当 $t=0$ 时无意义;而如果取 t 趋近于零,也不能得到好结果[9-12]。所以,这个解不能是问题(1)的任何正规解的表达式。鉴于正规解的重要性,就要研究其正规解的形式,以及其为正规时自由项所需要满足的条件。

1 形式解的积分变数替换

采用变数替换,将表达式(2)中前面一个积分做下列变数替换

$$x_0=x+2a\sqrt{t}\alpha,y_0=y+2a\sqrt{t}\beta,z_0=z+2a\sqrt{t}\gamma$$

而后面一个积分做下列变数替换

$$x_0=x+2a\sqrt{t-\tau}\alpha,y_0=y+2a\sqrt{t-\tau}\beta$$
$$z_0=z+2a\sqrt{t-\tau}\gamma$$

即可得

$$u=u(x,y,z,t)=\left(\frac{1}{\sqrt{\pi}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}g(x+2a\sqrt{t}\alpha,$$
$$y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x+2a\sqrt{t-\tau}\alpha,y+$$
$$2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma \quad (3)$$

2 解的正规性

定理 1 对于扩散方程 Cauchy 问题(1),在定解区域,如果其中

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1^0 函数 $g(x,y,z)$ 满足条件:

$$g(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\times\exp(-\alpha^2-\beta^2-\gamma^2)$$

及其关于变数 t 的偏导数,以及它关于变数 x,y,z 的各个二阶偏导数均一致连续,关于变数 α,β,γ 绝对可积,且使得成立下列三个极限等式

$$\lim_{\alpha\to\infty}[g_1(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

$$\lim_{\beta\to\infty}[g_2(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

$$\lim_{\gamma\to\infty}[g_3(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

2^0 函数 $f(x,y,z,t)$ 满足条件:

$$f(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times\exp(-\alpha^2-\beta^2-\gamma^2)$$

及其关于变数 t 的偏导数,以及关于变数 x,y,z 的各个二阶偏导数均一致连续,关于变数 α,β,γ,τ 绝对可积,且使得成立下列三个极限等式

$$\lim_{\alpha\to\infty}[f_1(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

$$\lim_{\beta\to\infty}[f_2(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

$$\lim_{\gamma\to\infty}[f_3(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\exp(-\alpha^2-\beta^2-\gamma^2)]=0$$

则表达式为式(3)所示的函数 $u=u(x,y,z,t)$ 是问题(1)正规解。

证明 在定解区域内,用 $u(x,y,z,t)$ 的表达式(3)计算。当 $t=0$ 时,有

$$u|_{t=0}=u(x,y,z,0)=\left(\frac{1}{\sqrt{\pi}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}g(x,y,z)\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+0=g(x,y,z)$$

用条件,交换微积分次序,化为累次积分,并用分部积分法,得

$$\left(\frac{\partial}{\partial t}-a^2\frac{\partial^2}{\partial x^2}-a^2\frac{\partial^2}{\partial y^2}-a^2\frac{\partial^2}{\partial z^2}\right)u(x,y,z,t)=$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\left(\frac{\partial}{\partial t}-a^2\frac{\partial^2}{\partial x^2}-a^2\frac{\partial^2}{\partial y^2}-a^2\frac{\partial^2}{\partial z^2}\right)$$

$$\Big[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}g(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t d\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x+2a\sqrt{t-\tau}\alpha,y+(2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma\Big]=$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\partial}{\partial t}-a^2\frac{\partial^2}{\partial x^2}-a^2\frac{\partial^2}{\partial y^2}-a^2\frac{\partial^2}{\partial z^2}\right)\times g(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y,z,t)\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t d\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\partial}{\partial t}-a^2\frac{\partial^2}{\partial x^2}-a^2\frac{\partial^2}{\partial y^2}-a^2\frac{\partial^2}{\partial z^2}\right)f(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma=\left(\frac{1}{\sqrt{\pi}}\right)^3\times$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[\frac{a\alpha}{\sqrt{t}}g_1(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)-a^2g_{11}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\Big]\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+\left(\frac{1}{\sqrt{\pi}}\right)^3\times$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[\frac{a\beta}{\sqrt{t}}g_2(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)-a^2g_{22}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\Big]\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+\left(\frac{1}{\sqrt{\pi}}\right)^3\times$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[\frac{a\gamma}{\sqrt{t}}g_3(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)-a^2g_{33}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\Big]\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+f(x,y,z,t)+$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t d\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[\frac{a\alpha}{\sqrt{t-\tau}}f_1(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-a^2f_{11}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\Big]\times\exp(-\alpha^2-\beta^2-\gamma^2)\,d\alpha d\beta d\gamma+$$

$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t d\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[\frac{a\beta}{\sqrt{t-\tau}}f_2(x+2a\sqrt{t-\tau}\alpha,$$

$$(y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-$$
$$a^2f_{22}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+$$
$$2a\sqrt{t-\tau}\gamma,\tau)]\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\frac{a\gamma}{\sqrt{t-\tau}}f_3(x+2a\sqrt{t-\tau}\alpha,\right.$$
$$y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-$$
$$a^2f_{33}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+$$
$$2a\sqrt{t-\tau}\gamma,\tau)]\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma=$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\times\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\alpha}{\sqrt{t}}g_1(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\right.\right.$$
$$-a^2g_{11}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)]\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\}\mathrm{d}\beta\mathrm{d}\gamma+\left(\frac{1}{\sqrt{\pi}}\right)^3\times$$
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\beta}{\sqrt{t}}g_2(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\right.\right.$$
$$-a^2g_{22}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)]\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\beta\}\mathrm{d}\alpha\mathrm{d}\gamma+\left(\frac{1}{\sqrt{\pi}}\right)^3\times$$
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\gamma}{\sqrt{t}}g_3(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)\right.\right.$$
$$-a^2g_{33}(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,z+2a\sqrt{t}\gamma)]\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)d\gamma\}\mathrm{d}\alpha\mathrm{d}\beta+f(x,y,z,t)+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\alpha}{\sqrt{t-\tau}}f_1(x+2a\sqrt{t-\tau}\alpha,y+\right.\right.$$
$$2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-$$
$$a^2f_{11}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+$$
$$2a\sqrt{t-\tau}\gamma,\tau)]\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\}\mathrm{d}\beta\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\beta}{\sqrt{t-\tau}}f_2(x+2a\sqrt{t-\tau}\alpha,y+\right.\right.$$
$$2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-$$
$$a^2f_{22}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+$$
$$2a\sqrt{t-\tau}\gamma,\tau)]\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\beta\}\mathrm{d}\alpha\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}\left[\frac{a\gamma}{\sqrt{t-\tau}}f_3(x+2a\sqrt{t-\tau}\alpha,y+\right.\right.$$
$$2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)-$$
$$a^2f_{33}(x+2a\sqrt{t-\tau}\alpha,y+2a\sqrt{t-\tau}\beta,z+$$
$$2a\sqrt{t-\tau}\gamma,\tau)]\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\gamma\}\mathrm{d}\alpha\mathrm{d}\beta=$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\left(\frac{-a}{2\sqrt{t}}\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[g_1(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,$$
$$z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]_{\alpha\to-\infty}^{\alpha\to+\infty}\mathrm{d}\beta\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\left(\frac{-a}{2\sqrt{t}}\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[g_2(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,$$
$$z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]_{\beta\to-\infty}^{\beta\to+\infty}\mathrm{d}\alpha\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\left(\frac{-a}{2\sqrt{t}}\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[g_3(x+2a\sqrt{t}\alpha,y+2a\sqrt{t}\beta,$$
$$z+2a\sqrt{t}\gamma)\exp(-\alpha^2-\beta^2-\gamma^2)]_{\gamma\to-\infty}^{\gamma\to+\infty}\mathrm{d}\alpha\mathrm{d}\beta+f(x,y,z,t)+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\frac{-a}{2\sqrt{t-\tau}}\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[f_1(x+2a\sqrt{t-\tau}\alpha,$$
$$y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)]_{\alpha\to-\infty}^{\alpha\to+\infty}\mathrm{d}\beta\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\frac{-a}{2\sqrt{t-\tau}}\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[f_2(x+2a\sqrt{t-\tau}\alpha,$$
$$y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)]_{\beta\to-\infty}^{\beta\to+\infty}\mathrm{d}\alpha\mathrm{d}\gamma+$$
$$\left(\frac{1}{\sqrt{\pi}}\right)^3\int_0^t\frac{-a}{2\sqrt{t-\tau}}\mathrm{d}\tau\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[f_3(x+2a\sqrt{t-\tau}\alpha,$$
$$y+2a\sqrt{t-\tau}\beta,z+2a\sqrt{t-\tau}\gamma,\tau)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)]_{\gamma\to-\infty}^{\gamma\to+\infty}\mathrm{d}\alpha\mathrm{d}\beta=f(x,y,z,t)$$

由此可见，表达式为式(3)的函数 $u=u(x,y,z,t)$ 满足问题(1)，所以它是问题(1)的正规解。证毕。

3　实例

求下列定解问题的解

$$\left.\begin{aligned}&\frac{\partial u}{\partial t}-\frac{\partial^2u}{\partial x^2}-\frac{\partial^2u}{\partial y^2}-\frac{\partial^2u}{\partial z^2}=0\\&u|_{t=0}=xyz\\&((x,y,z)\in R^3,0<t<+\infty)\end{aligned}\right\}\qquad(4)$$

解：用公式(3)，其中代以

$$a=1,f(x,y,z,t)=0,g(x,y,z)=xyz$$

得解

$$u=\left(\frac{1}{\sqrt{\pi}}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x+2a\sqrt{t}\alpha)\times$$
$$(y+2a\sqrt{t}\beta)(z+2a\sqrt{t}\gamma)\times$$
$$\exp(-\alpha^2-\beta^2-\gamma^2)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma=xyz$$

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Solution and Its Normality of Three Dimensional Diffusion of Cauchy Problem

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Abstract: The three-dimensional diffusion equations of Cauchy problem is studied. First one form solution is introduced, and then using variable replacement method, another form solution is gotten. Finally, a condition that this form solution is normal solution is put forward. This result mainly facilitates the theoretical analysis and application.

Key words: diffusion equation; Cauchy problem conditions; Fourier transformation; Laplace transformation; normality