第 24 卷第 5 期　　四川理工学院学报(自然科学版)　　Vol. 24　No. 5
2011 年 10 月　　Journal of Sichuan University of Science & Engineering(Natural Science Edition)　　Oct. 2011
文章编号:1673-1549(2011)05-0575-05

一类梁的横向振动方程的稳定性分析

张志军[1], 吕海炜[2], 刘启宽[1], 李映辉[2], 秦　营[1]

(1. 成都信息工程学院数学学院, 成都　610225;2. 西南交通大学力学与工程学院, 成都　610031)

摘　要:研究了一类梁的横向振动方程的稳定性态,使用 Galerkin 截断得到仅含时间变量的非线性常微分方程组。利用微分方程稳定性理论,讨论了系统平衡点的稳定性态。通过对参数的分析,得到了系统不同类型平衡点的存在条件,并给出了数值模拟。

关键词:振动;稳定性;平衡点;极限环

中图分类号:O175.12　　**文献标识码**:A

工程中许多问题都可以归结为梁,如电线杆、电视发射塔、风力机塔筒等,但目前对这些结构振动特性的计算都采用有限元方法,如何快速、准确计算这类结构的振动特性(如固有频率、模态等),是一个具有工程意义的课题。本文利用数学、力学中的振动分析理论和定性稳定性理论,对如下一类梁的横向振动方程的稳定性进行了研究:

$$\ddot{w}-\frac{\eta H}{\rho(H+h)(1-v_j^2)}\left(2\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x\partial t}+\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x\partial t}\right)+2V_e\frac{\partial^2 w}{\partial x\partial t}+\frac{\eta H}{12\rho(H+h)(1-V_j^2)}\frac{\partial^5 w}{\partial x^4\partial t}+\frac{E(h^3+3H^2h+3Hh^2)}{12\rho(H+h)(1-V_c^2)}\frac{\partial^4 w}{\partial x^4}+\frac{E_0H^3}{12\rho(H+h)(1-V_j^2)}\frac{\partial^4 w}{\partial x^4}+V_e^2\frac{\partial^2 w}{\partial x^2}-\frac{3Eh}{2\rho(H+h)(1-V_c^2)}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}-\frac{3E_0H}{2\rho(H+h)(1-V_j^2)}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}=0 \qquad (1)$$

1　平衡点性态分析

本文只考虑关于 w 的情况,并令 V_c, V_j 均为0,整理得

$$\ddot{w}+2V_e\frac{\partial^2 w}{\partial x\partial t}+\frac{\eta H}{12\rho(H+h)}\frac{\partial^5 w}{\partial x^4\partial t}+\left(\frac{E(h^3+3H^2h+3Hh^2)}{12\rho(H+h)}+\frac{E_0H^3}{12\rho(H+h)}\right)\frac{\partial^4 w}{\partial x^4}+V_e^2\frac{\partial^2 w}{\partial x^2}-\frac{3(Eh+E_0H)}{2\rho(H+h)}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}-\frac{\eta H}{\rho(H+h)}\left(2\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+\left(\frac{\partial w}{\partial x}\right)^2\right)\frac{\partial^2 w}{\partial x\partial t}=0 \qquad (2)$$

设(2)式有如下级数形式的解

$$w=\sum\varphi_i(x)q_i(t)=\varphi_1(x)q_1(t)+\varphi_2(x)q_2(t)+\cdots \qquad (3)$$

将(3)式代入(2)式整理得:

$$\sum\varphi_i(x)\ddot{q}_i(t)+2V_e\sum\varphi'_i(x)\dot{q}_i(t)+\frac{\eta H}{12\rho(H+h)}\sum\varphi_i''''(x)\dot{q}_i(t)+\left(\frac{E(h^3+3H^2h+3Hh^2)}{12\rho(H+h)}+\frac{E_0H^3}{12\rho(H+h)}\right)\sum\varphi_i''''(x)q_i(t)+V_e^2\sum\varphi_i''(x)q_i(t)-\frac{3(Eh+E_0H)}{2\rho(H+h)}\sum\varphi'_i(x)\varphi'_j(x)\varphi_k''(x)q_i(t)q_j(t)q_k(t)-\frac{\eta H}{\rho(H+h)}\left(2\sum\varphi'_i(x)\varphi_j''(x)\varphi'_k(x)q_i(t)q_j(t)\dot{q}_k(t)+\sum\varphi'_i(x)\varphi'_j(x)\varphi'_k(x)q_i(t)q_j(t)\dot{q}_k(t)\right)=0 \qquad (4)$$

将(4)式两边同乘 $\varphi_m(x)$, 并在 $[0,a]$ 上对 x 积分,再令

$$\int_0^a\varphi_i(x)\varphi_m(x)dx=a_{mi}$$

$$\int_0^a\varphi_m(x)\varphi'_i(x)dx=b_{mi}$$

收稿日期:2011-07-13

基金项目:国家自然科学基金项目(11072204);四川省应用基础研究计划项目(2010JY0079);成都信息工程学院自然科学发展基金项目(CSRF200601)

作者简介:张志军(1986-),男,山东济宁人,硕士生,主要从事微分方程及其应用方面的研究,(E-mail) zzj2270294@163.com

$$\int_0^a \varphi_m(x)\varphi_i''(x)dx = c_{mi},$$

$$\int_0^a \varphi_m(x)\varphi_i'''(x)dx = d_{mi}$$

$$\int_0^a \varphi_m(x)\varphi'_i(x)\varphi_j(x)\varphi_k''(x)dx = e_{mijk}$$

$$\int_0^a \varphi'_i(x)\varphi'_j(x)\varphi'_k(x)\varphi_m(x)dx = f_{mijk}$$

整理得:

$$\sum a_{mi}\ddot{q}_i(t) + \sum\left[2V_e b_{mi} + \frac{\eta H}{12\rho(H+h)}d_{mi}\right]\dot{q}_i(t) + \sum\left[V_e^2 c_{mi} + \left(\frac{E(h^3+3H^2h+3Hh^2)}{12\rho(H+h)} + \frac{E_0H^3}{12\rho(H+h)}\right)d_{mi}\right]q_i(t) - \sum\frac{3(Eh+E_0H)}{2\rho(H+h)}e_{mijk}q_i(t)q_j(t)q_k(t) - \sum\left[\frac{\eta H}{\rho(H+h)}(2e_{mikj}+f_{mijk})\right]q_i(t)q_j(t)\dot{q}_k(t) = 0 \quad (5)$$

根据实际生产的需求,取 $w(x,t) = \sin\frac{\pi}{a}xq_1(t)$,代入(5)式得

$$\ddot{q}_1 + \frac{\eta H\pi^4\dot{q}_1}{12\rho(H+h)} - \left[\frac{\pi^2}{a^2}V_e^2 - \frac{\pi^4(E(h^3+3H^2h+3Hh^2)+E_0H^3)}{12\rho(H+h)a^4}\right]q_1 + \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}q_1^3 + \frac{\eta H\pi^4}{2\rho(H+h)a^4}q_1^2\dot{q}_1 = 0 \quad (6)$$

令 $\dot{q}_1 = p_1$,则(6)式化为如下微分方程组

$$\begin{cases}\dot{q}_1 = -\frac{\eta H\pi^4}{12\rho(H+h)a^4}p_1 + \left[\frac{\pi^2}{a^2}V_e^2 - \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^4}{a^4}\right]q_1 - \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}q_1^3 - \frac{\eta H\pi^4}{2\rho(H+h)a^4}q_1^2p_1 \triangleq P(p_1,q_1) \\ \dot{q}_1 = p_1 \triangleq Q(p_1,q_1)\end{cases} \quad (7)$$

由 $\begin{cases}P(p_1,q_1) = 0\\ Q(p_1,q_1) = 0\end{cases}$ 得

$p_1 = 0$, $q_1 = 0$ 或

$$q_1 = \pm\sqrt{\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}V_e^2 - \frac{2[E(h^3+3H^2h+3Hh^2)+E_0H^3]}{9(Eh+E_0H)}}$$

显然,当 $V_e^2 > \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,系统(7)有三个平衡点:

$O(0,0)$

$$M_1\left(0,\sqrt{\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}V_e^2 - \frac{2[E(h^3+3H^2h+3Hh^2)+E_0H^3]}{9(Eh+E_0H)}}\right)$$

$$M_2\left(0,-\sqrt{\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}V_e^2 - \frac{2[E(h^3+3H^2h+3Hh^2)+E_0H^3]}{9(Eh+E_0H)}}\right)$$

当 $V_e^2 \leqslant \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,系统(7)有唯一平衡点 $O(0,0)$。

定理 1

(1)当 $V_e^2 > \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,平衡点 $O(0,0)$ 为系统(7)的鞍点。

(2) 当 $V_e^2 < \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,

① 若 $\Psi > 0$,平衡点 $O(0,0)$ 为系统(7)稳定的结点;

② 若 $\Psi < 0$,平衡点 $O(0,0)$ 为系统(7)稳定的焦点;

③ 若 $\Psi = 0$,平衡点 $O(0,0)$ 为系统(7)稳定的退化结点

$$\Psi = \left[\frac{\eta H\pi^4}{12\rho(H+h)a^4}\right]^2 + \frac{4\pi^2}{a^2}V_e^2 - \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{3\rho(H+h)}\cdot\frac{\pi^4}{a^4}$$

证 明

系统(7)在 $O(0,0)$ 的 Jacobi 矩阵为

$$\begin{pmatrix}-\frac{\eta H\pi^4}{12\rho(H+h)a^4} & \frac{\pi^2}{a^2}V_e^2 - \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^4}{a^4}\\ 1 & 0\end{pmatrix}$$

$$\begin{cases}p = \frac{\eta H\pi^4}{12\rho(H+h)a^4} > 0 \quad q = -\frac{\pi^2}{a^2}V_e^2 + \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^4}{a^4}\\ p^2 - 4q = \left[\frac{\eta H\pi^4}{12\rho(H+h)a^4}\right]^2 + \frac{\pi^2}{a^2}V_e^2 - \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{3\rho(H+h)}\cdot\frac{\pi^4}{a^4}\end{cases}$$

上述矩阵简写为 $\begin{pmatrix}-p & -q\\ 1 & 0\end{pmatrix}$,所以,

(1) 当 $V_e^2 > \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,$q < 0$,则平衡点 $O(0,0)$ 为系统(7)的鞍点。

(2) 当 $V_e^2 < \frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,$q > 0$。

① 若 $\Psi > 0$,则 $p^2 - 4q > 0$, $v_1 > 0$ 所以平衡点 $O(0,0)$ 为系统(7)稳定的结点。

② 若 $\Psi < 0$,则 $p^2 - 4q < 0$,$\lambda_{1,2} = \frac{-p \pm \sqrt{4q-p^2}i}{2}$ 所以平衡点 $O(0,0)$ 为系统(7)稳定的焦点。

③ 若 $\Psi = 0$，则 $p^2 - 4q = 0$，$\lambda_{1,2} = \frac{-p}{2} < 0$，$\lambda_{1,2}$ 对应的特征向量为 $\left(-\frac{p}{2} \quad 1\right)^T$，所以平衡点 $O(0,0)$ 为系统(7)稳定的退化结点。

定理2　当 $V_e^2 = \frac{E(h^3 + 3H^2h + 3Hh^2) + E_0H^3}{12\rho(H+h)} \cdot \frac{\pi^2}{a^2}$ 时，平衡点 $O(0,0)$ 为系统(7)的汇。

证明　当 $V_e^2 = \frac{E(h^3 + 3H^2h + 3Hh^2) + E_0H^3}{12\rho(H+h)} \cdot \frac{\pi^2}{a^2}$ 时，$q = 0$，$\lambda_1 = -p < 0$，$\lambda_2 = 0$，此时系统(7)变为：

$$\begin{cases} \bar{q}_1 = -\frac{\eta H\pi^4}{12\rho(H+h)a^4}p_1 - \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}q_1^3 \\ \quad -\frac{\eta H\pi^4}{2\rho(H+h)a^4}q_1^2p_1 \\ \bar{q}_1 = p_1 \end{cases} \tag{8}$$

作非退化线性变换

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{\eta H\pi^4}{12\rho(H+h)a^4} \\ 1 & 0 \end{pmatrix}\begin{pmatrix} p_1 \\ q_1 \end{pmatrix}$$

即

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{12\rho(H+h)a^4}{\eta H\pi^4} & -\frac{12\rho(H+h)a^4}{\eta H\pi^4} \end{pmatrix}\begin{pmatrix} u_1 \\ v_1 \end{pmatrix}$$

系统(8)变为：

$$\begin{cases} \ddot{u}_1 = \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3(u_1+v_1)^3 + \\ \quad \frac{\eta H\pi^4}{2\rho(H+h)a^4}\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2(u_1+v_1)^2v_1 \\ \dot{u}_1 = -\frac{\eta H\pi^4}{12\rho(H+h)a^4}v_1 - \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3 \\ \quad (u_1+v_1)^3 - \frac{\eta H\pi^4}{2\rho(H+h)a^4}\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2(u_1+v_1)^2v_1 \end{cases} \tag{9}$$

中心流形

$$w_{uc}(0,0) = \{(u_1,v_1) \mid v_1 = h(u_1), h(0) = h'(0) = 0\}$$

将 $h(u_1)$ 表示为幂级数

$$h(u_1) = c_2u_1^2 + c_3u_1^3 + o(u_1^4) \tag{10}$$

代入

$$h'(u_1)\bar{u}_1 - \bar{v}_1 = 0 \tag{11}$$

得

$$(2c_2u_1 + 3c_3u_1 + o(u_1))\left[\frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3(u_1 + c_2u_1 + c_3u_1 + o(u_1))^3 + \frac{\eta H\pi^4}{2\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2 (u_1 + c_2u_1 + c_3u_1 + o(u_1))^2(c_2u_1 + c_3u_1 + o(u_1))\right] + \frac{\eta H\pi^4}{12\rho(H+h)a^4}(c_2u_1 + c_3u_1 + o(u_1)) + \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3 (u_1 + c_2u_1 + c_3u_1 + o(u_1))^3 + \frac{\eta H\pi^4}{2\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2 (u_1 + c_2u_1 + c_3u_1 + o(u_1))^2(c_2u_1 + c_3u_1 + o(u_1)) = 0 \tag{12}$$

比较两边的各项系数得

$$c_2 = 0, c_3 = \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^4 > 0$$

故

$$h(u_1) = \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^4 u_1^3 + o(u_1^4)$$

于是中心流形 $w_{uc}(0,0)$ 上的解满足

$$\dot{u}_1 = \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3 u_1^3 + o(u_1^4)$$

在中心流形上的解，当 $u_1 > 0$ 时，$\dot{u}_1 < 0$，即当 $t \to +\infty$ 时，$u_1(t) \to 0$；当 $u_1 < 0$ 时，$\dot{u}_1 > 0$，即当 $t \to +\infty$ 时，$u_1(t) \to 0$。

求稳定流形

$$w_{uc}(0,0) = \{(u_1,v_1) \mid u_1 = \bar{h}(v_1), \bar{h}(0) = \bar{h}'(0) = 0\}$$

设

$$\bar{h}(v_1) = d_2v_1 + d_3v_1 + o(v_1) \tag{13}$$

代入

$$\bar{h}'(v_1)\bar{v}_1 - \bar{u}_1 = 0 \tag{14}$$

得

$$(2d_2v_1 + 3d_3v_1 + o(v_1))\left[-\frac{\eta H\pi^4}{12\rho(H+h)a^4}v_1 - \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^3 (v_1 + d_2v_1 + d_3v_1 + o(v_1))^3 - \frac{\eta H\pi^4}{2\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2(v_1 + d_2v_1 + d_3v_1 + o(v_1))^2v_1\right] - \frac{3(Eh+E_0H)\pi^4}{8\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right) 3(v_1 + d_2v_1 + d_3v_1 + o(v_1))^3 - \frac{\eta H\pi^4}{2\rho(H+h)a^4} \cdot \left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2(v_1 + d_2v_1 + d_3v_1 + o(v_1))^2v_1 = 0 \tag{15}$$

比较两边的各项系数得

$$d_2 = 0$$

$$d_3=\left[\frac{(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}\cdot\left(\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2-2\right]\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2$$

故

$$\bar{h}(v_1)=\left[\frac{(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}\cdot\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2-2\right]\left(-\frac{12\rho(H+h)a^4}{\eta H\pi^4}\right)^2v_1+o(v_1)$$

于是中心流形 $w_{wc}(0,0)$ 上的解满足

$$\bar{v}_1=-\frac{\eta H\pi^4}{12\rho(H+h)a^4}v_1+o(v_1)$$

当 $v_1>0$ 时, $\bar{v}_1<0$,即当 $t\to+\infty$ 时, $v_1(t)\to0$ 。

当 $v_1<0$ 时, $\bar{v}_1>0$,即当 $t\to+\infty$ 时, $v_1(t)\to0$ 。

综上知:原点是汇。

定理 3 当 $V_e^2>\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,

(1) 若 $\Lambda\geqslant0$,平衡点 M_1,M_2 是系统(7)稳定的结点。

(2) 若 $\Lambda<0$,平衡点 M_1,M_2 是系统(7)稳定的焦点。

$$\Lambda=\left(-\frac{\eta H\pi^4}{12\rho(H+h)a^4}-\frac{\eta H\pi^4}{2\rho(H+h)a^4}\cdot\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}\left[v_e-\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}\right]\right)^2-\frac{8\pi^2}{a^2}\left[v_e-\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}\right]$$

证明 当 $V_e^2>\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}$ 时,系统(7)有平衡点 M_1,M_2 ,二者有相同的 *Jacobi* 矩阵:

$$\begin{pmatrix}-\frac{\eta H\pi^4}{12\rho(H+h)a^4}v_1-\frac{\eta H\pi^4}{2\rho(H+h)a^4}\cdot\left(\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}\cdot v_e-\frac{2[E(h^3+3H^2h+3Hh^2)+E_0H^3]}{9(Eh+E_0H)}\right) & 1\\ \frac{\pi^2}{a^2}\cdot v_e-\left(\frac{E(h^3+3H^2h+3Hh^2)}{12\rho(H+h)}+\frac{E_0H^3}{12\rho(H+h)}\right)\frac{\pi^4}{a^4} & \\ -\frac{9(Eh+E_0H)\pi^4}{8\rho(H+h)a^4}\cdot\left(\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}\cdot v_e-\frac{2[E(h^3+3H^2h+3Hh^2)+E_0H^3]}{9(Eh+E_0H)}\right) & 0\end{pmatrix}^T$$

$$\lambda_1\cdot\lambda_2=\frac{2\pi^2}{a^2}\cdot v_e-\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{6\rho(H+h)}\cdot\frac{\pi^4}{a^4}=\frac{2\pi^2}{a^2}\left[v_e-\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}\right]>0$$

$$\lambda_1+\lambda_2=-\frac{\eta H\pi^4}{12\rho(H+h)a^4}-\frac{\eta H\pi^4}{2\rho(H+h)a^4}\cdot\frac{8\rho(H+h)a^2}{3(Eh+E_0H)\pi^2}\left[v_e-\frac{E(h^3+3H^2h+3Hh^2)+E_0H^3}{12\rho(H+h)}\cdot\frac{\pi^2}{a^2}\right]<0$$

则 $\lambda_1<0,\lambda_2<0$ 。

所以(1)若 $\Lambda\geqslant0$,平衡点 M_1,M_2 是系统(7)稳定的结点。

(2)若 $\Lambda<0$,平衡点 M_1,M_2 是系统(7)稳定的焦点。

2 数值模拟

系统简记为 $\begin{cases}\bar{p}_1=a_1p_1+a_2q_1a_3q_1^3+a_4q_1^2p_1\\ \bar{q}_1=p_1\end{cases}$,系统相图如图 1 ~ 图 4 所示。

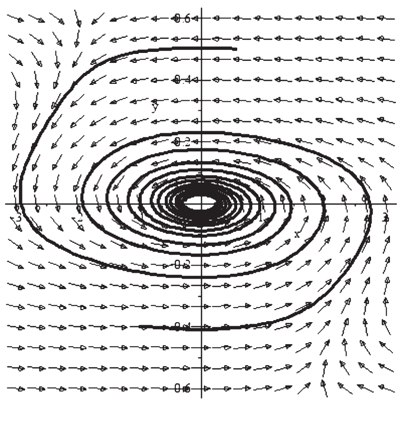

图 1 $a_1=-1,a_2=-100,a_3=-50,a_4=-100$ 系统相图

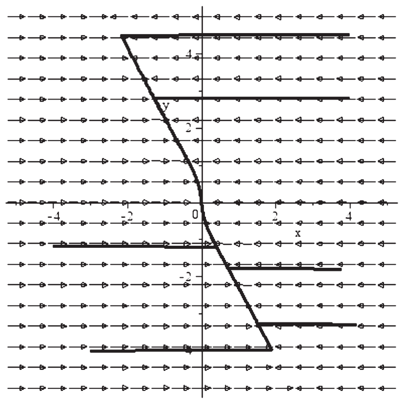

图 2 $a_1=-100,a_2=-10,a_3=-50,a_4=-100$ 系统相图

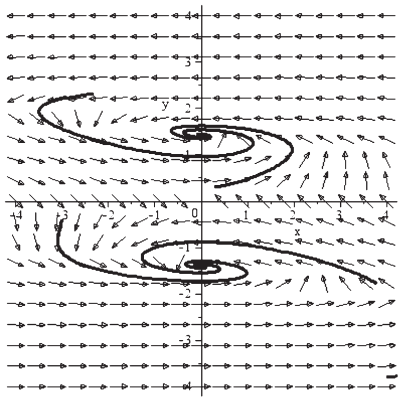

图 3 $a_1=-1, a_2=10, a_3=-5, a_4=-1$ 系统相图

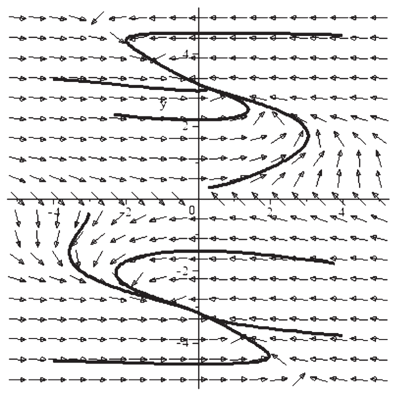

图 4 $a_1=-1, a_2=10, a_3=-1, a_4=-1$ 系统相图

3 结 论

(1)使用 Galerkin 截断得到仅含时间变量的非线性常微分方程组。

(2)利用微分方程稳定性理论,讨论了系统平衡点的稳定性态,通过对参数的分析,得到了系统不同类型平衡点的存在条件。

(3)对理论推导进行了数值模拟。

(4)系统的极限环问题还有待于研究。

参 考 文 献:

[1] 刘启宽,张兆强,陈 冲.一类具有功能反应的食饵-捕食模型的定性分析[J].重庆理工大学学报:自然科学版,2010,24(1):118-122.

[2] 张兆强,刘 皓,刘启宽.一类食饵为 Smith 增长模型且基于比率的食饵-捕食模型的定性分析[J].成都信息工程学院学报,2010,25(8):421-426.

[3] 刘启宽,刘 皓,李映辉.一类平面微分系统的定性分析[J].吉首大学学报:自然科学版,2010,31(4):18-23.

[4] 张大克,王玉杰.梁振动方程的一种新解法[J].工科数学,1999,15(2):46-50.

[5] 许士菊,王长华.梁振动方程的一个稳定的有限差分近似[J].吉林化工学院学报,2007,24(1):79-81.

[6] 周良强,陈予恕,陈芳启.梁振动方程的多参数高精度格式[J].安庆师范学院学报:自然科学版,2010,16(2):63-65.

[7] 张清泉,李映辉,姚 进.形状记忆合金梁动力稳定性及混沌运动[J].四川大学学报:工程科学版,2004,36(5):30-34.

[8] 张芷芬,丁同仁,黄文灶.微分方程定性理论[M].北京:科学出版社,1985.

Stability Analysis about the Transverse Vibration Equation of a Beam

ZHANG Zhi-jun[1], *LV Hai-wei*[2], *LIU Qi-kuan*[1], *LI Ying-hui*[2], *QIN Ying*[1]

(1. College of Mathematics, Chengdu University of Information Technology, Chengdu 610225, China;

2. School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, China)

Abstract: The stability condition of the transverse vibration equation about a beam is studied. By using Galerkin truncation, the nonlinear ordinary differential equations only containing the variable time is gotten. We discuss the stability of the system's equilibrium by using of differential equation stability theory. Besides, we can get the existence conditions of different types of the system's equilibrium through the analysis of parameters. The numerical simulation result is also given.

Key words: vibration; stability; equilibrium; limit cycle