

三角拟合解振荡微分方程的修正的预估校正方法

郑娟

(枣庄学院数学与统计学院, 山东 枣庄 277160)

摘要:以一类修正的预估校正 Adams 方法为基础,构造了新的三角拟合方法,给出新方法的局部截断误差,同时对新方法作了稳定性分析。数值实验的结果表明这个新方法较原始的修正的预估校正 Adams 方法及其它一些常用的方法在处理振荡问题时具有明显的高效性。

关键词:三角拟合;预估校正;振荡微分方程

中图分类号:O242

文献标识码:A

近年来,在天体力学、电子、物理化学、动量力学、材料科学等多个领域的一阶微分方程

$$y'(x) = f(x, y), y(x_0) = y_0 \tag{1}$$

的初值问题,尤其当它的解具有一定的周期性时一直备受人们关注。

在求它的数值解时三角拟合技术是其中常用的方法,Gautschi^[1]与 Lyche^[2]给出此项技术的理论基础,后来 Vanden Bergh^[3,4]及 Simos^[5-7]等对指数拟合、三角拟合 Runge - Kutta 方法作了深入的研究,Psihoyios^[8-10]对三角拟合预估校正 Adams 方法作了一系列研究,得出了很好的结果。本文以一类修正的预估校正 Adams 方法^[11]为基础,得到一个解具周期性的微分方程的三角拟合方法。

1 三角拟合修正的预估校正 Adams 方法

对方程(1)的数值解的问题,考虑下面形式的修正的预估校正 Adams 格式:

$$\begin{aligned} \overline{y_{k+1}} &= y_k + h(a_1 f(x_k, y_k) + a_2 f_{k-1} + a_3 f_{k-2} + a_4 f_{k-3}) \\ \overline{\overline{y_{k+1}}} &= y_k + h(b_1 f(x_{k+1}, \overline{y_{k+1}}) + b_2 f_k + b_3 f_{k-1} + b_4 f_{k-2}) \\ y_{k+1} &= c_1 \overline{y_{k+1}} + c_2 \overline{\overline{y_{k+1}}} \end{aligned} \tag{2}$$

让(2)式精确积分 $\{ \cos(\omega x), \sin(\omega x) \}$ 的线性组合,假定:

$$a_1 = \frac{55}{24}, a_2 = -\frac{59}{24}, a_3 = \frac{37}{24}, a_4 = -\frac{9}{24}$$

$$b_1 = \frac{9}{24}, b_2 = \frac{19}{24}, b_3 = -\frac{5}{24}, b_4 = \frac{1}{24}$$

可得方程组:

$$\begin{cases} -\frac{37}{24}c_2 \sin(2u)u + \frac{55}{64}c_1 u^2 + \frac{37}{64}c_1 \cos(2u)u^2 - \\ \frac{59}{64}c_1 \cos(u)u^2 - \frac{9}{64}c_1 \cos(3u)u^2 - \frac{1}{24}c_1 \sin(2u)u + \\ \frac{5}{24}c_1 \sin(u)u + \frac{59}{24}c_2 \sin(u)u + \frac{3}{8}c_2 \sin(3u)u + \\ \cos(u) - c_1 - c_2 = 0 \\ \frac{5}{24}c_1 \cos(u)u - \frac{37}{64}c_1 \sin(2u)u^2 + \frac{59}{64}c_1 \sin(u)u^2 + \\ \frac{9}{64}c_1 \sin(3u)u^2 + \frac{3}{8}c_2 \cos(3u)u + \sin(u) - \\ \frac{37}{24}c_2 \cos(2u)u - \frac{7}{6}c_1 u - \frac{55}{24}c_2 u - \frac{1}{24}c_1 \cos(2u)u + \\ \frac{59}{24}c_2 \cos(u)u = 0 \end{cases} \tag{3}$$

其中 $u = \omega h$ 。解方程组(3)得到 c_1, c_2 , 然后把 c_1, c_2 泰勒展开可得:

$$\begin{aligned} c_1 &= \frac{251}{270} + \frac{1574827}{2721600}u^2 + \frac{4529137397}{3919104000}u^4 + \\ &\frac{1068490995581039}{434550251520000}u^6 + \\ &\frac{4386039836587555517}{8134780708454400000}u^8 + \end{aligned}$$

$$c_2 = \frac{974982188632609897499189}{81998589541220352000000}u^{10} + \frac{52692485398110871883371832603}{2007325471969074216960000000}u^{12} + O(u^{13}) - \frac{19}{270} - \frac{1574827}{2721600}u^2 - \frac{4529137397}{3919104000}u^4 - \frac{1113153943017839}{434550251520000}u^6 - \frac{46024282453751465917}{8134780708454400000}u^8 - \frac{1024514187356554895559989}{81998589541220352000000}u^{10} - \frac{55384283104578443136259314203}{2007325471969074216960000000}u^{12} + O(u^{14}) \quad (4)$$

将(4)式所给系数的三角拟合修正的预估校正 Adams 方法记为 ModADMPCF1, 该方法的局部截断误差为:

$$LTE = \frac{53281}{518400}(y_n^{(6)} + \omega^6 y_n)h^6 + O(h^7) \quad (5)$$

因此这个方法的代数阶为5。根据(4)式, 因为 $u = \omega h$, 可知当 $\omega \rightarrow 0$ 时, 所得到的新方法 ModADMPCF1 将变为原始的修正的预估校正 Adams 方法(2)式。

2 稳定性分析

将得到的新方法应用到试验方程:

$$y' = \lambda y \quad (6)$$

可得差分方程:

$$\begin{aligned} & \left(1 + \frac{299}{240}h\lambda + \frac{2761}{3456}h^2\lambda^2\right)y_n - y_{n+1} - \\ & \left(\frac{14809}{17280}h^2\lambda^2 + \frac{11}{30}h\lambda\right)y_{n-1} + \\ & \left(\frac{9287}{17280}h^2\lambda^2 + \frac{53}{360}h\lambda\right)y_{n-2} - \\ & \left(\frac{251}{1920}h^2\lambda^2 + \frac{19}{720}h\lambda\right)y_{n-3} = 0 \end{aligned}$$

由边界轨迹法^[12]可画出时的绝对稳定性区域。图1与图2分别为原始方法及三角拟合新方法时的绝对稳定性区域。

3 数值实验

这里选用4种方法比较, 评价的标准是比较4种方法的全局误差及计算所用的函数的个数: 四阶 Adams 线性多步方法^[11], 用 ADAM4 表示。五阶 Adams 线性多步方法^[11], 用 ADAM5 表示。修正的预估校正 Adams 方法^[11], 用 ModADMP 表示。三角拟合修正的预估校正

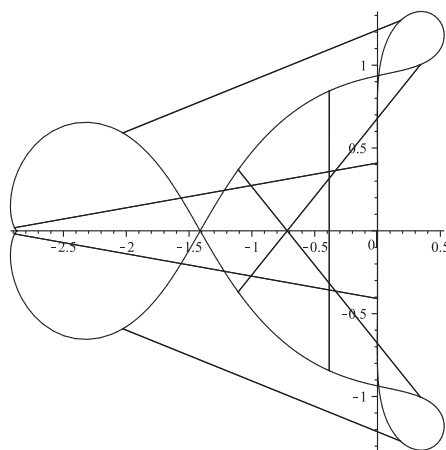


图1 原始方法的绝对稳定性区域

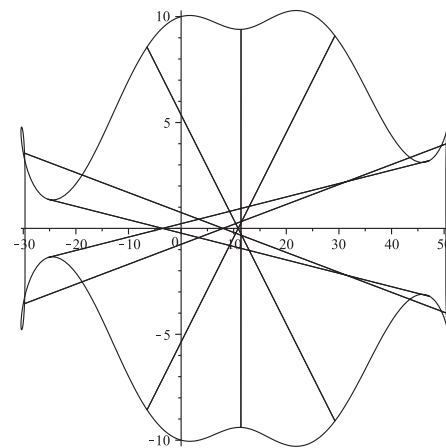


图2 新方法 $u = 10$ 时的绝对稳定性区域

Adams 方法, 即本文所得到的新方法, 用 ModADMPCF1 表示。

问题一 考虑非齐次方程:

$$\begin{aligned} y'' &= -100y + 99\sin(x) \\ y(0) &= 1, y'(0) = 11 \end{aligned} \quad (7)$$

其解析解为 $y(x) = \cos(10x) + \sin(10x) + \sin(x)$ 。对方程(7)做数值实验, 选择区间 $0 \leq x \leq 100$, 三角拟合新方法 ModADMPCF1 选择 $\omega = 10$ 。图3给出数值演示结果。

问题二 考虑面周期性初值问题:

$$\begin{aligned} y'' &= -81y + 80\sin(x) \\ y(0) &= \frac{1}{4}, y'(0) = 1 \end{aligned} \quad (8)$$

其解析解为 $y(x) = \frac{\cos(9x)}{4} + \sin(x)$ 。对方程(8)做数值实验, 选择区间 $0 \leq x \leq 100$, 三角拟合新方法 ModADMPCF1 选择 $\omega = 9$ 。图4给出数值演示结果。

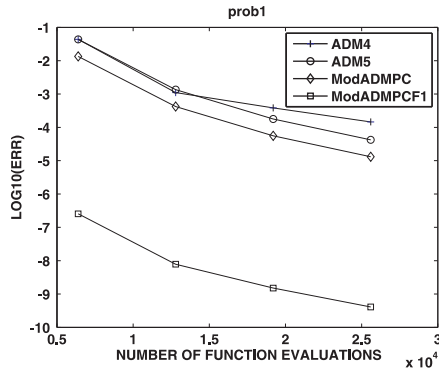


图 3 问题一的数值演示结果

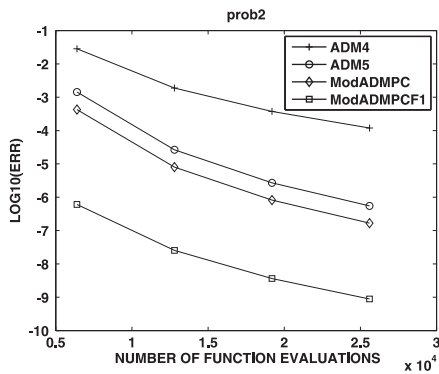


图 4 问题二的数值演示结果

问题三 考虑周期性初值问题:

$$y'' = -169y + 120\sin(x)$$

$$y(0) = \frac{3}{2}, y'(0) = 0 \tag{9}$$

其解析解为 $y(x) = \frac{\cos(13x)}{2} + \cos(7x)$ 。对方程(9)做数值实验,选择区间 $0 \leq x \leq 100$,三角拟合新方法 ModADMPCF1 选择 $\omega = 13$ 。图 5 给出数值演示结果。

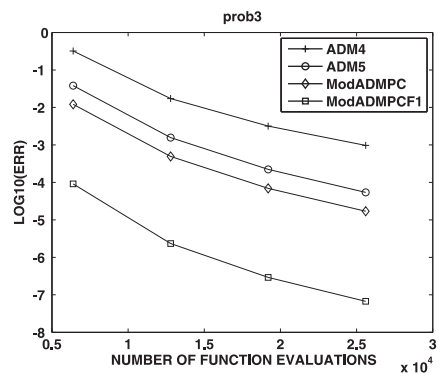


图 5 问题三的数值演示结果

问题四 考虑周期性初值问题:

$$y'' = -169y + (480 - 160i)e^{3ix}$$

$$y(0) = 4 + i, y'(0) = -23 + 22i \tag{10}$$

它等价于

$$u''(x) + 169u(x) - 480\cos(3x) - 160\sin(3x) = 0$$

$$v''(x) + 169v(x) - 480\sin(3x) + 160\cos(3x) = 0$$

这里 $y(x) = u(x) + iv(x)$ 。其解析解为

$$u(x) = -2\sin(13x) + \cos(13x) + 3\cos(3x) + \sin(3x)$$

$$v(x) = \sin(13x) + 2\cos(13x) + 3\sin(3x) - \cos(3x)$$

对方程(10)做数值实验,选择区间 $0 \leq x \leq 100$,三角拟合新方法 ModADMPCF1 选择 $\omega = 13$ 。图 6 给出数值演示结果。

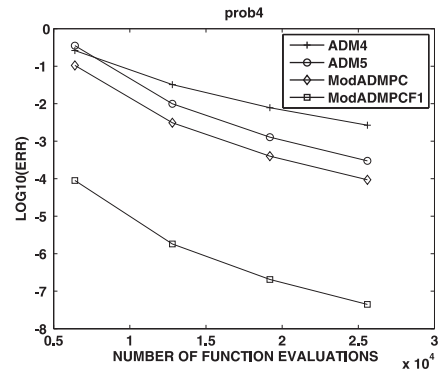


图 6 问题四的数值演示结果

4 结束语

以一类修正的预估校正 Adams 方法为基础,构造了新的三角拟合方法,这个新方法可以精确积分函数 $\{\cos(\omega x), \sin(\omega x)\}$ 的线性组合。数值实验的结果表明这个新方法较原始方法在处理振荡问题时具有明显的高效性,具有一定的实际应用价值。

参考文献:

- [1] Gautschi W. Numerical integration of ordinary differential equations based on trigonometric polynomials [J]. Numerische Mathematik, 1961:381-397.
- [2] Lyche T. Chebyshevian multistep methods for ordinary differential equations [J]. Numerische Mathematik, 1972 (19):65-75.
- [3] Vanden Berghe G, VanDaele M, Exponentially-fitted Numerov methods [J]. Journal of Computational and Applied Mathematics, 2007(200):140-153.
- [4] Vanden Berghe G, De Meyer H, Van Daele M, et al. Exponentially fitted Runge-Kutta methods [J]. Journal of Computational and Applied Mathematics, 2000(125):107-115.
- [5] Anastassi Z A, Simos T E. A trigonometrically fitted Runge-Kutta method for the numerical solution of orbital

- problems, *New Astronomy*[J].2005(10):301-309.
- [6] Anastassi Z A, Simos T.E. Trigonometrically fitted fifth-order Runge-Kutta methods for the numerical solution of the Schrödinger equation[J]. *Mathematical and Computer Modelling*,2005(12):877-886.
- [7] Simos T E. Exponentially fitted Runge-Kutta methods for the numerical solution of the Schrödinger equation and related problems [J]. *Computational Materials Science*, 2000(18):315-332.
- [8] Psihoyios G, Simos T E. Trigonometrically fitted predictor-corrector methods for IVPs with oscillating solutions [J]. *Journal of Computational and Applied Mathematics*,2003(158):135-144.
- [9] Psihoyios G, Simos T E. A fourth algebraic order trigonometrically fitted predictor-corrector scheme for IVPs with oscillating solutions [J]. *Journal of Computational and Applied Mathematics*,2005(175):137-147.
- [10] Psihoyios G, Simos T E. A new trigonometrically-fitted sixth algebraic order P-C algorithm for the numerical solution of the radial Schrödinger equation, *Mathematical and Computer Modelling*[J].2005(42):887-902.
- [11] Lambert J D. *Computational methods in ordinary differential equations* [M]. New York: John Wiley & Sons Inc,1973.
- [12] Lambert J D. *Numerical methods for ordinary differential systems: the initial value problem*[M]. New York: John Wiley & Sons, Inc.1991.
- [9] Psihoyios G, Simos T E. A fourth algebraic order trigono-

Trigonometrically Fitted Modified Predictor-corrector Scheme for IVPs with Oscillating Solutions

ZHENG Juan

(School of Mathematics and Statistics, Zaozhuang University, Zaozhuang 277160, China)

Abstract: A scheme of trigonometrically fitted method which is based on the fifth order modified is constructed a predictor – corrector method, the LTE of the new method is analyzed, and the region of absolute stability of our new method is presented. Some numerical experiments show the new trigonometrically fitted method is more efficient than the original method and some well – known methods used for the numerical solution of initial value problems with oscillating solutions.

Key words: trigonometric fitting; predictor-corrector methods; IVPs with oscillating solutions