文章编号: 1673-1549(2010)03-0270-05

含扩散项时滞模糊 Cohen-Grossberg 神经网络的指数稳定性

周 杰

(四川理工学院理学院, 四川 自贡 643000)

摘 要: 文章讨论了一类含扩散项的混合时滞模糊 Cohen-Grossberg神经网络, 通过一些基本分析手段和不等式技巧, 得到了平衡点的唯一性和全局指数稳定性的充分条件, 通过实例模型验证了指数稳定性。

关键词: 时滞; 扩散; Cohen-Grossberg 指数稳定性

中图分类号: O175.26　　文献标识码: A

引 言

Cohen-Grossberg神经网络是一种重要的神经网络模型, 最早由 Cohen-Grossberg建立[1]。在随后的二十多年里, 许多学者对这一模型进行了深入研究, 得到了模型稳定的充分条件[2-12]。在生物和人工神经网络中, 电子的扩散现象是不可避免的, 含扩散项的模型往往在一定程度上可以用来描述和刻画这些现象[2,4]。同时, 由于模糊逻辑在图像处理等问题中非常有用, 一些学者将模糊逻辑引入了神经网络中[5-7]。

受他们工作的启发, 本文将模糊逻辑引入 Cohen-Grossberg神经网络中, 考虑含扩散项的混合时滞模糊 Cohen-Grossberg神经网络, 通过不等式分析技巧和反证思想[9-10], 得到了平衡点的唯一性和全局指数稳定性的充分条件。

1 含扩散项的混合时滞模糊 Cohen-Grossberg神经网络

考虑含扩散项的混合时滞模糊 Cohen-Grossberg神经网络:

$$\left.\begin{aligned}&\frac{\partial u_i(t,x)}{\partial t}=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)-d_i(u_i(t,x))(c_i(u_i(t,x))-\sum_{j=1}^{n}a_{ij}f_j(u_j(t,x))-\sum_{j=1}^{n}b_{ij}\int_{-\infty}^{t}k_{ij}(t-s)g_j(u_j(s,x))ds+J_i-\\&\qquad\sum_{j=1}^{n}h_{ij}\mu_j-\wedge_{j=1}^{n}\alpha_{ij}f_j(u_j(t-\tau_{ij}(t),x))-\vee_{j=1}^{n}\beta_{ij}f_j(u_j(t-\tau_{ij}(t),x))-\wedge_{j=1}^{n}T_{ij}\mu_j-\vee_{j=1}^{n}H_{ij}\mu_j)\\&(x,t)\in\Omega\times[0,+\infty)\\&u_i(t,x)=0, x\in\partial\Omega\\&u_i(s,x)=\varphi_i(s,x),(s,x)\in(-\infty,0]\times\Omega\end{aligned}\right\}\tag{1}$$

其中 $i=1,2,\ldots,n$, n是神经网络神经元的个数; $x=(x_1,x_2,\ldots,x_m)^T\in\Omega\subset R^m$; $\Omega=\{x=(x_1,x_2,\ldots,x_m)^T \| |x_k|<l_k, k=1,2,\ldots,m\}$是一个带光滑边界 $\partial\Omega$且在 R^l中满足 Ω的测度大于 0的紧集; $u_i(t,x)$是第 i个神经元在时刻 t和空间 x的状态; 光滑函数 $D_{ik}=D_{ik}(t,x)\geqslant 0$是第 i个神经元的传输扩散算子; $d_i(u_i(t,x))$是放大器函数; $c_i(u_i(t,x))$代表运行函数; a_{ij}, b_{ij}是模糊反馈模块; h_{ij}是模糊前馈模块, $f_j(u_j(t,x))$, $g_j(u_j(t,x))$是第 j个神经元在时刻 t和空间 x的激活函数; k_{ij}是定义在 $[0,+\infty)$上的非负连续实值函数且满足 $\int_0^{\infty}k_{ij}(s)ds=1$; $\tau_{ij}(t)$是在 t时刻信号从第 i个神经元传输到第 j个神经元的时滞且满足 $0\leqslant\tau_{ij}(t)\leqslant\tau$; μ_j, J_j分别表示第 i个神经元的输入和干扰; α_{ij}, β_{ij}分别表示

收稿日期: 2010-02-24

基金项目: 四川理工学院 2008年人才引进项目(2008RCYJ10)

作者简介: 周杰(1982-), 男, 四川成都人, 助教, 硕士, 主要从事微分方程稳定性方面的研究。

模糊反馈最小模块和模糊反馈最大模块; T_{ij}, H_{ij} 分别表示模糊前馈最小模块和模糊前馈最大模块; $\wedge$, $\vee$ 分别表示模糊与运算和模糊或运算; $\varphi_i(s,x)(i=1,2,\ldots,n)$ 在 $(-\infty,0]\times\Omega$上有界连续。

定义 1　称常向量 $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T$ 是系统(1)的平衡点, 如果 u^* 满足

$$c_i(u_i^*)-\sum_{j=1}^n a_{ij}f_j(u_j^*)-\sum_{j=1}^n b_{ij}g_j(u_j^*)+J_i-\sum_{j=1}^n h_{ij}\mu_j-\wedge_{j=1}^n\alpha_{ij}f_j(u_j^*)-\vee_{j=1}^n\beta_{ij}f_j(u_j^*)-\wedge_{j=1}^nT_{ij}\mu_j-\vee_{j=1}^nH_{ij}\mu_j=0 \tag{2}$$

本文中, 始终假设系统(1)存在连续解 $u(t,x)=(u_1(t,x),u_2(t,x),\ldots,u_n(t,x))^T\in R^n$ 和平衡点 $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T\in R^n$。

定义 2　方程(1)的平衡点 $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T\in R^n$ 称为是全局指数稳定的, 如果存在 $\varepsilon>0$, $\forall t>0$, 有

$$\|u(t,x)-u^*\|_2\leqslant\|\varphi-u^*\|_2e^{-\varepsilon t} \tag{3}$$

引理 1[8]　令 Ω是满足 $|x_i|<l_i(i=1,2,\ldots,m)$的超立方, $h(x)\in C^1(\Omega)$是一个实值函数, 在边界上满足 $h(x)|_{\partial\Omega}=0$, 则 $\int_\Omega h^2(x)dx\leqslant l_i^2\int_\Omega\left|\frac{\partial h}{\partial x_i}\right|^2dx$。

引理 2[7]　对任意的 u^1和 u^2, 有

$$|\wedge_{j=1}^n\alpha_{ij}f_j(u_j^1)-\wedge_{j=1}^n\alpha_{ij}f_j(u_j^2)|\leqslant\sum_{j=1}^n|\alpha_{ij}||f_j(u_j^1)-f_j(u_j^2)|$$

$$|\vee_{j=1}^n\beta_{ij}f_j(u_j^1)-\vee_{j=1}^n\beta_{ij}f_j(u_j^2)|\leqslant\sum_{j=1}^n|\beta_{ij}||f_j(u_j^1)-f_j(u_j^2)|$$

为了得到我们的主要结果, 在本文中, 我们假设:

(A_1)激活函数 f_j和 g_j 是李谱希兹连续的, 即有常数 $F_j>0$和 $G_j>0$, 使得对于任意的 $\zeta_1,\zeta_2\in R$, $|f_j(\zeta_1)-f_j(\zeta_2)|\leqslant F_j|\zeta_1-\zeta_2|$, $|g_j(\zeta_1)-g_j(\zeta_2)|\leqslant G_j|\zeta_1-\zeta_2|$成立。

(A_2) $c_i(\cdot)$ 是连续的, 存在常数 $\overline{c_i}$ 使得 $\frac{c_i(s_1)-c_i(s_2)}{s_1-s_2}=\overline{c_i}>0$, $i=1,2,\ldots,n$, $s_1,s_2\in R$。

(A_3)存在常数 $\underline{d_i}$和 $\overline{d_i}$满足 $0<\underline{d_i}\leqslant d_i(u_i)\leqslant\overline{d_i}$, $i=1,2,\ldots,n$。

(A_4)核函数 $k_{ij}:[0,+\infty)\rightarrow[0,+\infty)(i,j=1,2,\ldots,n)$是实值非负连续函数且满足:

(1) $\int_0^\infty k_{ij}(s)ds=1$。

(2)存在 $\varepsilon>0$使得 $\int_0^\infty e^{\varepsilon s}k_{ij}(s)ds<+\infty$。即存在常数 δ_i使得 $\int_0^\infty e^{\varepsilon s}k_{ij}(s)ds\leqslant\delta_i$成立。

2　主要结果

定理 1　假设条件(A_1)、(A_2)和(A_4)成立, 且满足:

$$|\overline{c_i}|-\sum_{j=1}^n|a_{ji}|F_i-\sum_{j=1}^n|b_{ji}|G_i-\sum_{j=1}^n|\alpha_{ji}|F_i-\sum_{j=1}^n|\beta_{ji}|F_i>0 \tag{4}$$

则方程(1)的平衡点 $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T$ 是唯一的。

证明　令 u^* 和 u^{**} 是(1)的任意两个平衡点, 则

$$\begin{aligned}0={}&(c_i(u_i^*)-c_i(u_i^{**}))-\Big(\sum_{j=1}^n a_{ij}f_j(u_j^*)-\sum_{j=1}^n a_{ij}f_j(u_j^{**})\Big)-\\&\sum_{j=1}^n b_{ij}\int_{-\infty}^t k_{ij}(t-s)(g_j(u_j^*)-g_j(u_j^{**}))ds-\\&(\wedge_{j=1}^n\alpha_{ij}f_j(u_j^*)-\wedge_{j=1}^n\alpha_{ij}f_j(u_j^{**}))-\\&(\vee_{j=1}^n\beta_{ij}f_j(u_j^*)-\vee_{j=1}^n\beta_{ij}f_j(u_j^{**}))\geqslant\\&|c_i(u_i^*)-c_i(u_i^{**})|-\sum_{j=1}^n|a_{ij}||f_j(u_j^*)-f_j(u_j^{**})|-\\&\sum_{j=1}^n|b_{ij}|\int_{-\infty}^t k_{ij}(t-s)|g_j(u_j^*)-g_j(u_j^{**})|ds-\\&|\wedge_{j=1}^n\alpha_{ij}f_j(u_j^*)-\wedge_{j=1}^n\alpha_{ij}f_j(u_j^{**})|-\\&|\vee_{j=1}^n\beta_{ij}f_j(u_j^*)-\vee_{j=1}^n\beta_{ij}f_j(u_j^{**})|\geqslant\\&|\overline{c_i}||u_i^*-u_i^{**}|-\sum_{j=1}^n|a_{ij}|F_j|u_j^*-u_j^{**}|-\\&\sum_{j=1}^n|b_{ij}|\int_{-\infty}^t k_{ij}(t-s)G_j|u_j^*-u_j^{**}|ds-\\&\sum_{j=1}^n|\alpha_{ij}|F_j|u_j^*-u_j^{**}|-\sum_{j=1}^n|\beta_{ij}|F_j|u_j^*-u_j^{**}|\geqslant\\&\Big(|\overline{c_i}|-\sum_{j=1}^n|a_{ji}|F_i-\sum_{j=1}^n|b_{ji}|G_i-\sum_{j=1}^n|\alpha_{ji}|F_i-\sum_{j=1}^n|\beta_{ji}|F_i\Big)\\&|u_i^*-u_i^{**}|\geqslant0\end{aligned}$$

所以, $u_i^*=u_i^{**}$, 即系统(1)的平衡点是唯一的。

为了方便起见, 记

$$\varepsilon_i=\sum_{k=1}^l\frac{2D_{ik}}{l_k^2}+2\overline{c_i}\,\underline{d_i}-\overline{d_i}\sum_{j=1}^n|a_{ij}|F_j-\overline{d_i}\sum_{j=1}^n|a_{ji}|F_i-\overline{d_i}\sum_{j=1}^n|b_{ij}|G_j-\overline{d_i}\sum_{j=1}^n|\alpha_{ij}|F_j-\overline{d_i}\sum_{j=1}^n|\beta_{ij}|F_j$$

$$\eta_i=\overline{d_i}\sum_{j=1}^n|\alpha_{ji}|F_i-\overline{d_i}\sum_{j=1}^n|\beta_{ij}|F_i$$

$$\sigma_i=\overline{d_i}\sum_{j=1}^n|b_{ji}|G_i,\quad i=1,2,\ldots,n$$

定理 2　假设$(A_1)-(A_4)$成立, 且系数满足:

$$\frac{\varepsilon_i}{\eta_i+\sigma_i\delta_i}>1,\ \forall i=1,2,\ldots,n \tag{5}$$

则系统(1)的平衡点 $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T$ 是全局指数稳定的。

证明　首先选取(A_4)中充分小的常数 ε满足 $0<\varepsilon<\varepsilon_i$, 使得

$$\frac{\varepsilon_i - \varepsilon}{\eta_i e^{\varepsilon\tau} + \sigma_i \delta_i} > 1, \forall i = 1, 2, \ldots, n \quad (6)$$

为了方便证明, 作一个变换 $y_i(t, x) = u_i(t, x) - u_i^*$, $\Phi_i(s, x) = \varphi_i(s, x) - u_i^*$, 则系统(1)变为:

$$\frac{\partial y_i(t, x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t, x)}{\partial x_k}) + d_i(y_i(t, x) + u_i^*)(-c_i(y_i(t, x) + u_i^*) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t, x) + u_j^*) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s) g_j(y_j(s, x) + u_j^*) ds + \wedge_{j=1}^{n} \alpha_{ij} f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) + \vee_{j=1}^{n} \beta_{ij} f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) + c_i u_i^* - \sum_{j=1}^{n} a_{ij} f_j(u_j^*) - \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s) g_j(u_j^*) ds - \wedge_{j=1}^{n} \alpha_{ij} f_j(u_j^*) - \vee_{j=1}^{n} \beta_{ij} f_j(u_j^*)) =$$
$$\sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t, x)}{\partial x_k}) + d_i(y_i(t, x) + u_i^*)((-c_i(y_i(t, x) + u_i^*) + c_i u_i^*) + \sum_{j=1}^{n} a_{ij}(f_j(y_j(t, x) + u_j^*) - f_j(u_j^*)) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s)(g_j(y_j(s, x) + u_j^*) - g_j(u_j^*)) ds + \wedge_{j=1}^{n} \alpha_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) + \vee_{j=1}^{n} \beta_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*))) \quad (7)$$

在(7)式两边同乘 $y_i(t, x)$, 再在 Ω上关于 x积分, 有

$$\int_\Omega y_i(t, x)\frac{\partial y_i(t, x)}{\partial t} dx = \int_\Omega y_i(t, x) \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t, x)}{\partial x_k}) dx + \int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \times ((-c_i(y_i(t, x) + u_i^*) + c_i u_i^*) + \sum_{j=1}^{n} a_{ij}(f_j(y_j(t, x) + u_j^*) - f_j(u_j^*)) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s)(g_j(y_j(s, x) + u_j^*) - g_j(u_j^*)) ds + \wedge_{j=1}^{n} \alpha_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) + \vee_{j=1}^{n} \beta_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) dx$$

由边界条件和引理 1得到

$$\frac{1}{2}\frac{d}{dt}\|y_i(t, x)\|_2^2 \leqslant -\sum_{k=1}^{l} \int_\Omega \frac{D_{ik}}{l_k^2} |y_i(t, x)|^2 dx + \int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \times ((-c_i(y_i(t, x) + u_i^*) + c_i u_i^*) + \sum_{j=1}^{n} a_{ij}(f_j(y_j(t, x) + u_j^*) - f_j(u_j^*)) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s)(g_j(y_j(s, x) + u_j^*) - g_j(u_j^*)) ds + \wedge_{j=1}^{n} \alpha_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) + \vee_{j=1}^{n} \beta_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) dx$$

则

$$\frac{d}{dt}\|y_i(t, x)\|_2^2 \leqslant -\sum_{k=1}^{l} \frac{2D_{ik}}{l_k^2}\|y_i(t, x)\|_2^2 + 2\int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*)(-\overline{c_i} y_i(t, x)) dx + 2\int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \sum_{j=1}^{n} a_{ij}(f_j(y_j(t, x) + u_j^*) - f_j(u_j^*)) dx + 2\int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s)(g_j(y_j(s, x) + u_j^*) - g_j(u_j^*)) ds dx + 2\int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \wedge_{j=1}^{n} \alpha_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) dx + 2\int_\Omega y_i(t, x)(d_i(y_i(t, x) + u_i^*) \vee_{j=1}^{n} \beta_{ij}(f_j(y_j(t - \tau_{ij}(t), x) + u_j^*) - f_j(u_j^*)) dx \leqslant$$
$$-\sum_{k=1}^{l} \frac{2D_{ik}}{l_k^2}\|y_i(t, x)\|_2^2 - 2\overline{c_i}\overline{d_i}\int_\Omega (y_i(t, x))^2 dx + 2\overline{d_i}\sum_{j=1}^{n} |a_{ij}| F_j \int_\Omega |y_i(t, x)||y_j(t, x)| dx + 2\overline{d_i}\int_\Omega \int_0^\infty |y_i(t, x)| \sum_{j=1}^{n} |b_{ij}| k_{ij}(\theta) G_j |y_j(t-\theta, x)| d\theta dx + 2\overline{d_i}\sum_{j=1}^{n} |\alpha_{ij}| F_j \int_\Omega |y_i(t, x)||y_j(t - \tau_{ij}(t), x)| dx + 2\overline{d_i}\sum_{j=1}^{n} |\beta_{ij}| F_j \int_\Omega |y_i(t, x)||y_j(t - \tau_{ij}(t), x)| dx \leqslant$$
$$-\sum_{k=1}^{l} \frac{2D_{ik}}{l_k^2}\|y_i(t, x)\|_2^2 - 2\overline{c_i}\overline{d_i}\|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |a_{ij}| F_j \|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |a_{ij}| F_j \|y_j(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |b_{ij}| G_j \|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |b_{ij}| G_j \int_0^\infty k_{ij}(\theta)\|y_j(t-\theta, x)\|_2^2 d\theta + \overline{d_i}\sum_{j=1}^{n} |\alpha_{ij}| F_j \|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |\alpha_{ij}| F_j \|y_j(t - \tau_{ij}(t), x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |\beta_{ij}| F_j \|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |\beta_{ij}| F_j \|y_j(t - \tau_{ij}(t), x)\|_2^2 \leqslant$$
$$-\sum_{k=1}^{l} \frac{2D_{ik}}{l_k^2} y_i(t, x)_2^2 - 2\overline{c_i}\overline{d_i}\|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |a_{ij}| F_j \|y_i(t, x)\|_2^2 + \overline{d_i}\sum_{j=1}^{n} |a_{ji}| F_i \|y_i(t, x)\|_2^2 +$$

$$\overline{d}_i\sum_{j=1}^{n}|b_{ij}|G_j\|y_i(t,x)\|_2^2+$$
$$\overline{d}_i\sum_{j=1}^{n}|b_{ji}|G_j\int_0^\infty k_{ji}(\theta)y_i(t-\theta,x)_2^2d\theta+$$
$$\overline{d}_i\sum_{j=1}^{n}|\alpha_{ij}|F_j\|y_i(t,x)\|_2^2+$$
$$\overline{d}_i\sum_{j=1}^{n}|\alpha_{ji}|F_i\|y_i(t-\tau_{ji}(t),x)\|_2^2+$$
$$\overline{d}_i\sum_{j=1}^{n}|\beta_{ij}|F_j\|y_i(t,x)\|_2^2+$$
$$\overline{d}_i\sum_{j=1}^{n}|\beta_{ji}|F_i\|y_i(t-\tau_{ji}(t),x)\|_2^2\leqslant$$
$$(-\sum_{k=1}^{l}\frac{2D_{ik}}{l_k^2}-2\overline{c}_i\overline{d}_i+\overline{d}_i\sum_{j=1}^{n}|a_{ij}|F_j+$$
$$\overline{d}_i\sum_{j=1}^{n}|a_{ji}|F_i+\overline{d}_i\sum_{j=1}^{n}|b_{ij}|G_j+\overline{d}_i\sum_{j=1}^{n}|\alpha_{ij}|F_j+$$
$$\overline{d}_i\sum_{j=1}^{n}|\beta_{ij}|F_j)\|y_i(t,x)\|_2^2+$$
$$\overline{d}_i\sum_{j=1}^{n}|b_{ji}|G_j\int_0^\infty k_{ji}(\theta)\|y_i(t-\theta,x)\|_2^2d\theta+$$
$$(\overline{d}_i\sum_{j=1}^{n}|\alpha_{ji}|F_i+\overline{d}_i\sum_{j=1}^{n}|\beta_{ji}|F_i)\|y_i(t-\tau_{ji}(t),x)\|_2^2\leqslant$$
$$-\varepsilon_i\|y_i(t,x)\|_2^2+\eta_i\|y_i(t-\tau_{ji}(t),x)\|_2^2+$$
$$\sigma_i\int_0^\infty k_{ji}(\theta)\|y_i(t-\theta,x)\|_2^2d\theta$$

那么

$$\frac{d}{dt}\|y_i(t,x)\|_2^2+\varepsilon_i\|y_i(t,x)\|_2^2\leqslant$$
$$\eta_i\|y_i(t-\tau_{ji}(t),x)\|_2^2+$$
$$\sigma_i\int_0^\infty k_{ji}(\theta)\|y_i(t-\theta,x)\|_2^2d\theta$$

则

$$e^{\varepsilon_i t}\|y_i(t,x)\|_2^2-\|y_i(0,x)\|_2^2\leqslant$$
$$\eta_i\int_0^t e^{\varepsilon_i s}\|y_i(s-\tau_{ji}(s),x)\|_2^2ds+$$
$$\sigma_i\int_0^t e^{\varepsilon_i s}\int_0^\infty k_{ji}(\theta)\|y_i(s-\theta,x)\|_2^2d\theta ds$$

即

$$\|y_i(t,x)\|_2^2\leqslant e^{-\varepsilon_i t}\|\phi_i\|_2^2+$$
$$G_ie^{-\varepsilon t}\int_0^t e^{\varepsilon s}\|y_i(s-\tau_{ji}(s),x)\|_2^2ds+$$
$$R_ie^{-\varepsilon t}\int_0^t e^{\varepsilon s}\int_0^\infty k_{ji}(\theta)y_i(s-\theta,x)_2^2d\theta ds$$

对任意的, 只需证

$$\|y_i(t,x)\|_2^2\leq\|\phi_i\|_2^2e^{-\varepsilon t},\tag{8}$$

$\forall t\geq 0,\ i=1,2,\ldots,n$

为了证明(8)式, 首先证明

$$\|y_i(t,x)\|_2^2<B\|\phi_i\|_2^2e^{-\varepsilon t},\tag{9}$$

$\forall t\geq 0,\ B>1,\ i=1,2,\ldots,n$

如果(9)式不成立, 则一定存在 $t_1>0$ 和某个 $i_0\in\{1,\ldots,n\}$, 使得

$$\|y_{i_0}(t_1,x)\|_2^2=B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}\tag{10}$$

和

$$\|y_{i_0}(t,x)\|_2^2\leq B\|\phi_{i_0}\|_2^2e^{-\varepsilon t},\ 0\leq t\leq t_1,\ B>1\tag{11}$$

由(A_1)-(A_4)、(10)式和(11)式有

$$\|y_{i_0}(t_1,x)\|_2^2\leq e^{-\varepsilon_{i_0}t_1}\|\phi_{i_0}\|_2^2+$$
$$e^{-\varepsilon_{i_0}t_1}G_{i_0}\int_0^t e^{\varepsilon s}B\|\phi_{i_0}\|_2^2e^{-\varepsilon(s-\tau_{ji_0}(s))}ds+$$
$$R_{i_0}e^{-\varepsilon_{i_0}t_1}\int_0^t e^{\varepsilon s}\int_0^\infty k_{ji_0}(\theta)B\|\phi_{i_0}\|_2^2e^{-\varepsilon(s-\theta)}d\theta ds=$$
$$e^{-\varepsilon_{i_0}t_1}\|\phi_{i_0}\|_2^2+$$
$$B\|\phi_{i_0}\|_2^2G_{i_0}e^{-\varepsilon_{i_0}t_1}\int_0^t e^{\varepsilon s}e^{-\varepsilon(s-\tau_{ji_0}(s))}ds+$$
$$R_{i_0}e^{-\varepsilon_{i_0}t_1}B\|\phi_{i_0}\|_2^2$$
$$\int_0^t e^{\varepsilon s}\int_0^\infty k_{ji_0}(\theta)e^{-\varepsilon(s-\theta)}d\theta ds=B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}[\frac{1}{B}e^{\varepsilon t_1-\varepsilon_{i_0}t_1}+$$
$$G_{i_0}e^{\varepsilon t_1-\varepsilon_{i_0}t_1}\int_0^t e^{\varepsilon s}e^{-\varepsilon(s-\tau_{ji_0}(s))}ds+$$
$$R_{i_0}e^{\varepsilon t_1-\varepsilon_{i_0}t_1}\int_0^t e^{\varepsilon s}\int_0^\infty k_{ji_0}(\theta)e^{-\varepsilon(s-\theta)}d\theta ds]\leq$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}[\frac{1}{B}e^{-(\varepsilon_{i_0}-\varepsilon)t_1}+G_{i_0}e^{-(\varepsilon_{i_0}-\varepsilon)t_1}$$
$$\int_0^t e^{\varepsilon s}e^{-\varepsilon s}e^{\varepsilon\tau}ds+R_{i_0}e^{-(\varepsilon_{i_0}-\varepsilon)t_1}\int_0^t e^{(\varepsilon_{i_0}-\varepsilon)s}D_{i_0}ds]=$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}\{\frac{1}{B}e^{-(\varepsilon_{i_0}-\varepsilon)t_1}+$$
$$\frac{G_{i_0}e^{\varepsilon\tau}}{\varepsilon_{i_0}-\varepsilon}[1-e^{-(\varepsilon_{i_0}-\varepsilon)t_1}]+\frac{R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}[1-e^{-(\varepsilon_{i_0}-\varepsilon)t_1}]\}=$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}\{[\frac{1}{B}-\frac{G_{i_0}e^{\varepsilon\tau}}{\varepsilon_{i_0}-\varepsilon}-\frac{R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}]$$
$$e^{-(\varepsilon_{i_0}-\varepsilon)t_1}+\frac{G_{i_0}e^{\varepsilon\tau}+R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}\}<$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}\{[1-\frac{G_{i_0}e^{\varepsilon\tau}+R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}]e^{-(\varepsilon_{i_0}-\varepsilon)t_1}+$$
$$\frac{G_{i_0}e^{\varepsilon\tau}+R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}\}<$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}\{[1-\frac{G_{i_0}e^{\varepsilon\tau}+R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}]+\frac{G_{i_0}e^{\varepsilon\tau}+R_{i_0}D_{i_0}}{\varepsilon_{i_0}-\varepsilon}\}=$$
$$B\|\phi_{i_0}\|_2^2e^{-\varepsilon t_1}.$$

这与(11)式矛盾, 所以(9)式成立。令 $B\to 1$, 则(8)式成立。

3 例 子

考虑含扩散项的模糊 Cohen-Grossberg神经网络模型(1), 其中 $i,j=1,2$ 取

$f_j(r)=g_j(r)=0.5(|r+1|-|r-1|)$

$d_2(u_2(t,x))=2+\cos u_2(t,x)$

$c_i(u_i(t,x))=3u_i(t,x)$

$a_{11}=a_{12}=a_{21}=a_{22}=0.1$

$b_{11} = b_{12} = b_{21} = b_{22} = 011$

$T_{11} = T_{12} = T_{21} = T_{22}$

$= H_{11} = H_{12} = H_{21} = H_{22} = 011$

$A_{11} = A_{12} = A_{21} = A_{22}$

$= B_{11} = B_{12} = B_{21} = B_{22} = 011$

$L = (L_1, L_2)^T = (1, 1)^T$

$8 = \{x \| x_k | < 1, k = 1\}$

$D_{11} = D_{21} = 2, k_{ij} = e^{-s}$

则 $f_j(r), g_j(r)$ 满足 (A_1), $F_j = G_j = 1$。经过简单计算, 得到 $E_1 = E_2 = 7, G_1 = G_2 = 112, R_1 = R_2 = 016$。$\frac{E_i}{G_i + R_i D_i} = \frac{7}{112 + 016 \# 112} > 1$, 满足(5)式。所以由定理 2可知, 系统(1)的平衡点是全局指数稳定的。

参 考 文 献:

[1] Cohen M A, Grossberg S. Absolute stability and global pattern formation and parallel memory storage by com2petitive neural networks[J]. IEEE Trans Syst Man Cybernet, 1983, 13: 8152826.

[2] Zhao H Y, Wang K L. Dynamical behaviors of Cohen2Grossberg neural networks with delays and reaction2diffusion terms[J]. Neurocomputing, 2006, 70: 5362543.

[3] Qiu J L. Exponential stability of impulsive neural net2works with time2varying delays and reaction2diffusion terms[J]. Neurocomputing, 2007, 70: 110221108.

[4] Zhou Q H, Wan L, Sun J H. Exponential stability of reaction2diffusion generalized Cohen2Grossberg neural networks with time2varying delays[J]. Chaos Solitons and Fractals, 2007, 32: 171321719.

[5] Yang T, Yang L B, Wu C W, et al. Fuzzy cellular neu2ral networks: theory[J]. In: Proc of IEEE interna2tional workshop on cellular neural networks and appli2cations, 1996: 1812186.

[6] Yang T, Yang L B, Wu C W, et al. Fuzzy cellular neu2ral networks: applications[J]. In: Proc of IEEE inter2national workshop on cellular neural networks and ap2plications, 1996: 2252230.

[7] Yang T, Yang L B. The global stability of fuzzy cellu2lar neural network[J]. IEEE Trans Circ Syst I, 1996, 43: 8802883.

[8] Lu J G. Global exponential stability and periodicity of reaction2diffusion delayed recurrent neural networks with Dirichlet boundary conditions[J]. Chaos, Soli2tons and Fractals, 2008, 35: 1162125.

[9] Zhou J, Li S Y, Yang Z G. Global exponential stability of Hopfield neural networks with distributed delays[J]. Applied Mathematical Modelling, 2009, 33: 151321520.

[10] Zhou J, Li S Y. Global exponential stability of impul2sive BAM neural networks with distributed delays[J]. Neurocomputing, 2009, 72: 168821693.

[11] 罗文品, 钟守铭. 脉冲时滞细胞神经网络系统的指数稳定性[J]. 四川理工学院学报:自然科学版, 2008, 21(4): 21223.

[12] 罗文品, 钟守铭, 杨军. 具有脉冲和时滞的细胞神经网络的全局指数稳定性[J], 四川理工学院学报:自然科学版, 2009, 22(3): 123.

Exponential Stability of Fuzzy Cohen2Grossberg Neural Networks with Mixed Delays and Reaction2diffusion

ZHOU Jie

(School of Science, Sichuan University of Science & Engineering, Zigong 643000, China)

Abstract: A fuzzy Cohen2Grossberg neural networks with mixed delays and reaction2diffusion are discussed. By some in2equality techniques, the uniqueness of equilibrium point of fuzzy Cohen2Grossberg neural networks with mixed delays and re2action2diffusion is proved. Then, a sufficient condition is given to ensure the global exponential stability of the equilibrium point for this model. An example was also worked out to demonstrate the application of our result.

Key words: time delays; reaction2diffusion; Cohen2Grossberg; exponential stability