

# Application of Portfolio Theory in the Stock Investment

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**Abstract:** Using statistics of ten industries in Shanghai Stock Exchange ( SSE ) , including Materials , Energy , Industry , Alternative Consumption , Consumption , Finance , Pharmacy , Information , Telecom and Public Use , study the portfolio theory of Markowitz. This paper is going to explain how the mean - variance model of Markowitz works , with the maximum and minimum expected return , the minimum variance determining the efficient frontier , then choose the reasonable portfolio according to the efficient frontier.

**Key words:** Portfolio theory; Stock investment; Expected return; Variance; Efficient frontier; Portfolio

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## Introduction

The beginning of modern finance is the 1950s , during this period , complex mathematical models were introduced into the investment and capital market. In 1952 , The American economist , financial expert Markowitz , who published < portfolio selection > , held the view that by using the relationship between mean and variance in statistics , one can establish the relationship between return and investment in investment. He mentioned , the investment risk consisted of systematic risk , which cannot be removed , and unsystematic risk , which can be eliminated through diversive investment. Investors can obtain the minimum risk in a certain return level , or the maximum return in a certain risk level through building a reasonable portfolio , making use of the covariance between assets and the quadratic programming model. The emergence of portfolio theory represented that the study of finance began to the quantitative direction , and the research progress is called "the first revolution of the 'Wall Street' . "

With the rapid development of global economy , financial mathematics not only greatly influences the operation of investment and capital market , but also has been widely used in financial markets <sup>[1]</sup>. All sorts of new financial products , financial derivatives and financial services , benefit from the vigorous development of global financial innova-

tions; those including portfolio choice , the investment decision analysis , the knowledge of financial market operation rule and risk analysis and management have played a crucial role in financial institutions , corporations and personal investment decisions<sup>[2]</sup>. This paper will focus on the application of portfolio theory in stock investment with detailed examples.

## 1 choose industries to invest

Markowitz's portfolio theory is based on a series of strict hypotheses , it is impossible to draw some conclusions without premises. This paper takes ten industries of Shanghai Stock Exchange ( SSE ) , including Materials , Energy , Industry , Alternative Consumption , Consumption , Finance , Pharmacy , Information , Telecom and Public Use as sample to carry out the study. Considering that the members of the above ten industries are the most representative and the highest liquid stocks , they are chosen to be the tools of analyzing the industry trends. The purpose of examples in this paper is selecting the industries to invest , but its methods can still be applied to certain stocks choice.

### 1. 1 premise of measuring the expected return

Investors have to follow the below five conditions<sup>[3-6]</sup>:  
(1) the stock market of Shanghai Stock Exchange( SSE) is effective , that is to say , the stock price can fully reflect available information , and investors can keep up with the

changes of share prices in the stock market. They understand the reasons for stock changes as well. Also, the stock price can correctly reflect the intrinsic value; (2) investment assets can be infinite divided, and investors can buy any part of the invested asset, if necessary, they can purchase equity; (3) investors are all rational ones, namely the risk aversion, seeking the maximum profit in a certain level of risk or the minimum risk in a certain level of return; (4) For the sake of the maximization of the expected wealth, investors are not allowed to bear and bull. (5) All investors are able to describe the returns and risk quantitatively, and investment decisions are based on the expected returns and expected risk of securities, and the expected risk is estimated by the expected rate of return, which follows normal distribution.

## 1.2 Find the correlation between all optional industries

Table1 the correlation coefficients between the ten industries

	public use	telecom	finance	pharmacy	alternative	materials	energy	industry	consumption	information
public use	1.0000									
telecom	0.4448	1.0000								
finance	0.9259	0.4873	1.0000							
pharmacy	-0.1228	0.7029	-0.0117	1.0000						
alternative	0.3006	0.8383	0.4079	0.8723	1.0000					
materials	0.5537	0.8892	0.6193	0.6332	0.8109	1.0000				
energy	0.7559	0.8109	0.8250	0.3846	0.6709	0.9204	1.0000			
industry	0.6950	0.8803	0.7105	0.4768	0.7314	0.9329	0.9375	1.0000		
consumption	0.0988	0.8391	0.1958	0.9620	0.9480	0.7854	0.5721	0.6566	1.0000	
information	-0.0398	0.7517	0.0530	0.9751	0.9003	0.6700	0.4308	0.5479	0.9622	1.0000

### 1.2.3 Choose weak correlated or negatively correlated industries

It can be seen from the graph that industry, energy and materials are highly related, the correlation coefficients are all higher than 0.9, and they are significant in 0.01 level, which is consistent with common sense, so materials is chosen as an object. Another which is worth mentioning is that alternative consumption and consumption belong to the same type of industry, and judging from the statistical results, they also satisfy the characteristics of highly correlated and significant in 0.01 level, so one of the two can remain in the portfolio. Compared with alternative consumption, the correlation coefficients between consumption and pharmacy, information, telecom reach above 0.75, which is higher than those of alternative consumption, so alternative consumption is kept to join the portfolio. In addition, both information and telecom belong to the communication industry, after comparing the two, it can be obtained that the correlation coefficients between information and alternative consumption, pharmacy reach more than 0.9 and significant in 0.01 level, therefore information is ex-

### 1.2.1 Select data

In this paper, data is selected from the statistics provided by the Shanghai Stock Exchange (SSE) from January 9<sup>th</sup>, 2009 to April 21<sup>st</sup>, 2011, with 554 trading days' information, including daily and weekly closing prices of ten industries. When analyzing the data, it is recognized that the daily returns have characteristics of small value and numerous quantity; thus, in the following analysis, weekly closing prices are used. There are 118 weeks in total.

### 1.2.2 Use STATA to calculate correlation coefficients and test significance.

Markowitz held the perspective that it is more possible to reduce the risk by choosing the industries which are weak correlated or negatively correlated with each other. Table 1 shows the output from statistical software STATA, with the correlation coefficients between the ten industries<sup>[7]</sup>.

cluded from the portfolio. The remaining industries are: materials, alternative consumption, pharmacy, finance, telecom, public use. Although among these six industries, the correlation coefficient between public use and finance reaches 0.9, the correlation coefficients between public use and other industries are low, therefore public use is reserved; furthermore, though there are some high relevance industries, their industry types are different, in order to ensure the integrity and diversity of the selected industries, they are all included in the portfolio.

### 1.2.4 Check whether the selected industries are normal distributed

Through the weekly return of the public use sector, a bar chart is drawn by EXCEL to show the frequency distribution. In figure 1, it can be clearly seen that returns between -2% and 3% have the largest number, and the frequency of less than 2% and greater than 3% decline in turn, which is consistent with the central limit theorem, therefore it is also accord with the characteristics of normal distribution. Similarly, the other five industries also follow

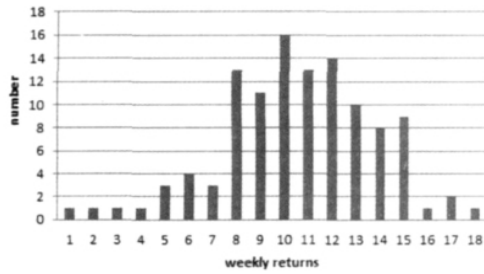


Figure 1 The frequency distribution of weekly returns of public use

similar frequency distributions. It can be concluded that the

Table2 the mean and variance weekly returns

	Public use	telecom	finance	pharmacy	alternative	materials
Mean	0.2716%	0.4900%	0.4575%	0.7731%	0.7700%	0.8455%
Variance	0.12%	0.18%	0.19%	0.16%	0.17%	0.24%

The expected return of the portfolio is:

$$E(r) = w_1E(r_1) + w_2E(r_2) + w_3E(r_3) + w_4E(r_4) + w_5E(r_5) + w_6E(r_6)$$

$E(r)$  is the expected return, and  $w_1, w_2, w_3, w_4, w_5, w_6$  are the proportions of public use, telecom, finance, pharmacy, alternative consumption and materials in the portfolio respectively.  $E(r_1), E(r_2), E(r_3), E(r_4), E(r_5), E(r_6)$

Table3 the covariance among industries

covariance	Public use	Telecom	Finance	Pharmacy	Alternative	Materials
Public use	1					
Telecom	0.0010040	1				
Finance	0.0010805	0.0008292	1			
Pharmacy	0.0007557	0.0010801	0.0002593	1		
Alternative	0.0011883	0.0013245	0.0009986	0.0012274	1	
Materials	0.0014363	0.0013431	0.0015875	0.0009988	0.001564	1

It can be obtain from Table 3 that the variance of portfolio is:

$$\begin{aligned} \sigma^2 = & (w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + w_4^2\sigma_4^2 + w_5^2\sigma_5^2 + w_6^2\sigma_6^2) + \\ & \left( 2w_1w_2cov_{1,2} + 2w_1w_3cov_{1,3} + 2w_1w_4cov_{1,4} + \right. \\ & \left. 2w_1w_5cov_{1,5} + 2w_1w_6cov_{1,6} \right. \\ & \left. 2w_2w_3cov_{2,3} + 2w_2w_4cov_{2,4} + \right. \\ & \left. 2w_2w_5cov_{2,5} + 2w_2w_6cov_{2,6} \right) + \\ & (2w_3w_4cov_{3,4} + 2w_3w_5cov_{3,5} + 2w_3w_6cov_{3,6}) + \\ & (2w_4w_5cov_{4,5} + 2w_4w_6cov_{4,6}) + \\ & (2w_5w_6cov_{5,6}) \end{aligned}$$

With  $\sigma^2$  represents the expected variance, and  $w_1, w_2, w_3, w_4, w_5, w_6$  represent the proportion of public use, telecom, finance, pharmacy, alternative consumption and materials in the portfolio respectively;  $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2$  represent the variance of public use, telecom, finance, pharmacy, alternative consumption and materials in the portfolio respectively;  $cov_{1,2}, cov_{1,3}, cov_{1,4}, cov_{1,5}, cov_{1,6}$  represent the covariances between public use and telecom, finance, pharmacy, alternative consumption and materials respec-

trading system is reliable and stable; the establishment of portfolio of the selected six industries is effective. Thus, the next phase of the test can be carried out<sup>[8]</sup>.

## 2 Establish mean - variance model

Markowitz used the share price in the portfolio as random variables, the mean and variance in Statistics as measurements of return and risk respectively. Through the test of correlation, six industries are chosen, after calculating the mean and variance of weekly returns, the results are displayed in Table 2.

are the means of weekly returns of public use, telecom, finance, pharmacy, alternative consumption and materials respectively.

Due to the calculation of the portfolio variance among industries needs to use the covariance between industries, so using EXCEL to calculate the covariance, the results are displayed in Table 3.

tively;  $cov_{2,3}, cov_{2,4}, cov_{2,5}, cov_{2,6}$  represent the covariances between telecom and finance, pharmacy, alternative consumption and materials respectively;  $cov_{3,4}, cov_{3,5}, cov_{3,6}$  represent the covariances between finance and pharmacy, alternative consumption and materials respectively;  $cov_{4,5}, cov_{4,6}$  represent the covariances between pharmacy and alternative consumption, materials respectively;  $cov_{5,6}$  represents the covariance between alternative consumption and materials.

## 3 determine the portfolio with the maximum and minimum expected return, and the minimum variance

The following calculation is based on the assumptions that there is no capital restriction, if in practical case, investors have the funds limit, just add a restraint in the programming solver<sup>[8]</sup>. In This paper, the programming solver function is offered by EXCEL, and gets the results in Table 4.

Table4 the portfolios with the maximum and minimum expected return and the minimum variance

portfolio	Public use	telecom	finance	pharmacy	alternative	materials	total	return	variance
1	0	0	0	0	0	100%	100%	0.0085	0.00240
2	31.75%	0	27.88%	40.37%	0	0	100%	0.0053	0.00098
3	100%	0	0	0	0	0	100%	0.0027	0.00117

portfolio 1 yields the highest return , namely invest 100% in materials , materials industry , and get a weekly return of 0.85% ; portfolio 3 yields the lowest return , namely invest 100% in public use industry , and get a weekly return of 0.27% ; portfolio 2 yields the lowest variance 0.98% , and get a weekly return of 0.53% .

#### 4 draw the efficient frontier

Markowitz thought that the efficient frontier is formed by combining the maximum and minimum return portfolio

and the minimum variance portfolio , the portfolios on the frontier are reasonable and available , the ones below the frontier are invalid<sup>[9]</sup> .

Between the minimum and maximum expected returns , set several expected returns , and get the corresponding portfolios which minimize the standard deviation. Also , using EXCEL programming solver function , 15 candidate portfolios are chosen , the detailed results are displayed in Table 5.

Table5 15 candidate portfolios

Portfolio	public use	telecom	finance	pharmacy	alternative	materials	total	return	variance	remark
1	0	0	0	0	0	100%	100%	0.0085	0.00240	Max. return
2	0	0	0	35.08%	0	64.92%	100%	0.0082	0.00167	
3	0	0	0	57.35%	5.13%	37.52%	100%	0.0080	0.00145	
4	0	0	7.14%	58.09%	7.49%	27.29%	100%	0.0077	0.00134	
5	0	0	12.37%	59.17%	6.03%	22.43%	100%	0.0075	0.00127	
6	0	0	20.25%	60.79%	3.84%	15.12%	100%	0.0072	0.00118	
7	0	0	25.49%	61.87%	2.38%	10.26%	100%	0.0070	0.00113	
8	0	0	39.00%	61.00%	0	0	100%	0.0065	0.00103	
9	11.01%	0	37.35%	51.64%	0	0	100%	0.0060	0.00099	
10	31.75%	0	27.88%	40.37%	0	0	100%	0.0053	0.00098	Min. var.
11	52.99%	0	18.18%	28.83%	0	0	100%	0.0045	0.00099	
12	66.98%	0	11.79%	21.23%	0	0	100%	0.0040	0.00102	
13	86.57%	0	2.84%	10.59%	0	0	100%	0.0033	0.00109	
14	94.34%	0	0	5.66%	0	0	100%	0.0030	0.00113	
15	100%	0	0	0	0	0	100%	0.0027	0.00117	Min. return

According to the above 15 groups of data , a scatter and line graph of return and risk is shown in Figure 2:

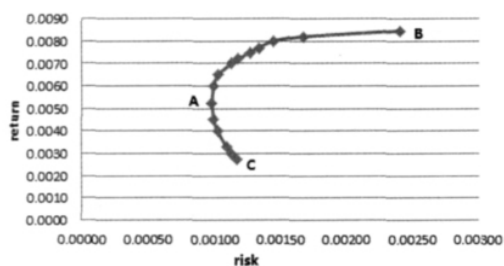


Figure 2 The efficient frontier of the portfolio

point A represents the minimum variance portfolio , point B , C represent the maximum and minimum return portfolios respectively. When connecting the scattered points by a smooth curve , the frontier of the portfolio can be obtained. It can be seen that portfolios 1 to 10 in the diagram are in the effective frontier AB , thus they belong to the optimized portfolios.

The points below arc AB either have lower return in the same risk level , or have higher risk in the same return level. These points do not satisfy the requirements of investors

seeking return maximization or risk minimization , so they are not in the consideration of rational investors.

#### 5 select certain portfolio

Completing the efficient frontier AB , investors determine their own portfolio according to their risk aversion. Usually , this can be done by combining with indifference curves. It is mentioned by Markowitz that individual optimal portfolio is the tangency point of indifference curves and efficient frontier<sup>[10]</sup> . Indifference curves indicate different reactions of investors to the variance and return level , the choices in the same indifference curve make no difference to them , no matter high - risk - high - return , or low - risk - low - return is same. According to Markowitz , all investors are rational , so the return is proportional to the risk , which means the slope is positive , and the lower right - side is to bulge. Moreover , the specific choice of each investor depends on the degree of risk aversion. In order to be more clear and specific , it can be explained with diagrams. The degree of risk aversion can be divided into high , moderate and low. Their indifference curves respectively are displayed in Figure 3.



Figure 3 The indifference curves of high , moderate and low risk aversion

In the same increased risk , a high risk aversion demands higher returns in correspond to increased risk , and a low risk aversion demands relatively lower returns in correspond to increased risk. Consequently , the slope of the indifference curve of high risk aversion is greater and steeper. This explains that investors will choose different portfolios according to the different perspectives of risk aversion.

## 6 Conclusion

Over the last 60 years' development of financial mathematics , although it appeared numerous comparatively better approaches to guide the investment decision - making , portfolio theory of Markowitz , which is one of the most important theories of financial mathematics , still plays an essential role , and it is the foundation of all developed theories , especially the mean - variance model. Apparently , the limitations of theoretical assumptions exist; however , it makes positive progress for the return weigh and risk diversification<sup>[8]</sup>. Hence , the investment decision - making of investors nowadays is supposed to be on the basis of portfolio theory , and uses various mathematical tools and methods to analyze the operation rule of securities market.

### References:

- [1] Duan Chong. The research review and prospect of financial mathematics [J]. Science. 2009( 24) : 442. ( In Chinese)
- [2] Zhang You-lan ,Zhou Ai-min. The research and progress of financial mathematics [J]. Studies in college mathematics. 2004 ( 4) : 53. ( In Chinese)
- [3] Cong Guo-hua ,Ye Yong-gang ,Wang Shu-zhong. Some deep understandings on financial mathematics [J]. Journal of Harbin Senior finance college. 2007 ( 2) : 57. ( In Chinese)
- [4] Liu Fei-fei ,You Gui-yun Ma Ke-wei ,The application of portfolio theory of Markowitz in securities market of China [J]. Economic Review. 2008 ( 2) : 29. ( In Chinese)
- [5] Liu Zhi-lin ,Deng Dai-jun ,Cao Hong. The analysis of modern portfolio theory [J]. Modern Business. 2010 ( 32) : 31. ( In Chinese)
- [6] Wu Xiu-jun. The view of Mathematical models [J]. Journal of Jingmen Vocational Technical College. 2001 ,( 6) : 85. ( In Chinese)
- [7] Yan Zhu-mei ,Liu Yi-wen ,Huang Jing-yin. The Application and its Model Development of Securities Investment Combination Theory in our Country [J]. ECONOMIC RESEARCH GUIDE. 2008 ( 14) : 104. ( In Chinese)
- [8] Liu Ying-pan ,Wang Yu-jing. The Application of Portfolio Theory in Futures Investment [J]. JOURNAL OF DALIAN UNIVERSITY. 2008( 4) : 116-119. ( In Chinese)
- [9] Xiong Chi. The research on portfolio theory [J]. LAO QU JIAN SHE. 2009( 14) : 24. ( In Chinese)
- [10] Yuan Jun. The review and prospect of financial mathematics [J]. Commercial Times. 2008 ( 13) : 68. ( In Chinese)

# 证券组合选择理论在股票投资中的应用

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**摘 要:** 以上海证券交易所中上证材料 ,上证能源 ,上证工业 ,上证可选( 可选消费) ,上证消费 ,上证金融 ,上证医药 ,上证信息 ,上证电信 ,上证公用十个行业的数据为样本 ,利用马科维茨证券组合选择理论 ,通过均值 - 方差模型 ,以期望收益最大值、最小值 ,方差最小值确定投资组合的有效边界 ,根据有效边界选择投资组合。

**关键词:** 证券组合选择理论; 股票投资; 收益期望; 方差; 边界; 投资组合